

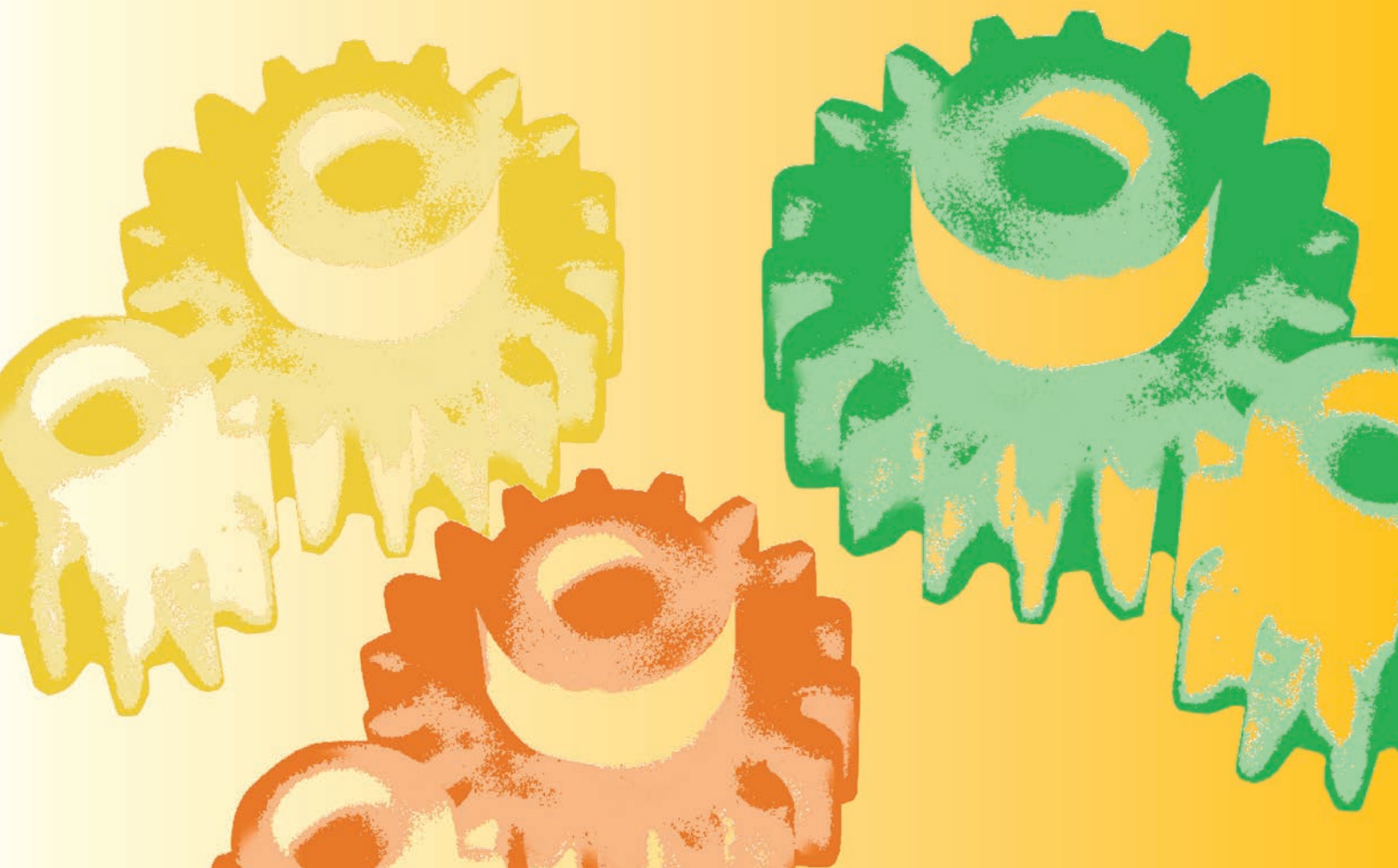
VOLUME 4

Rudolf Och

# GEARS

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**FRENCO**

# Preface

The field of gears and splines is varied and extensive. Gearings are characterised in that teeth of any shape are radially arranged on a cylinder. The term gearing is usually only used in the technical field if the teeth are of functional importance.

This book is a compilation of individual documentaries created over a period of 40 years with the experience of the author. It was completely revised before printing and reflects the state of the art.

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# The author

Graduate Engineer (Dipl. Ing., FH) Rudolf Och was born in Bamberg, Germany in 1951. After graduating in mechanical engineering he founded FRENCO GmbH in Nuremberg, Germany in 1978. In the beginning, the company only engaged in the development and manufacture of spline gauges. Over the years, however, the business was extended to include the full spectrum of gear and spline metrology. This development is supported by numerous inventions.

The author was a member of the American Standards Institute for Splines ANSI and has been Chairman of the German standards committee AA 2.1 since 1993. During the chairmanship, the German term for spline (Passverzahnung) was officially introduced and all relevant German standards were revised. The international standard ISO 4156 was also completely revised under German leadership by the responsible standards committee ISO/TC 14.

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# 1. Gear Pair

Running gears transmit torque from one axis to another. The driving and the driven gear axes are always parallel to each other. The tooth flanks of a gear pair roll off each other without any gliding motion. This is possible because of the involute shape of the tooth flanks. There are other flank forms beside the involute shape, for example cycloids, but they are rarely used. The involute is by far the most commonly used flank form. This document only deals with gear pairs with straight teeth.

## 1.1 Rolling behaviour

A gear pair functions in a similar manner to a rope drive with a driving and a driven rope drum. Both, speed-increase and speed-reduction ratios are possible. The centre distance is fixed but can be modified in its design. The gear ratio is determined by the diameter of the two rope drums.

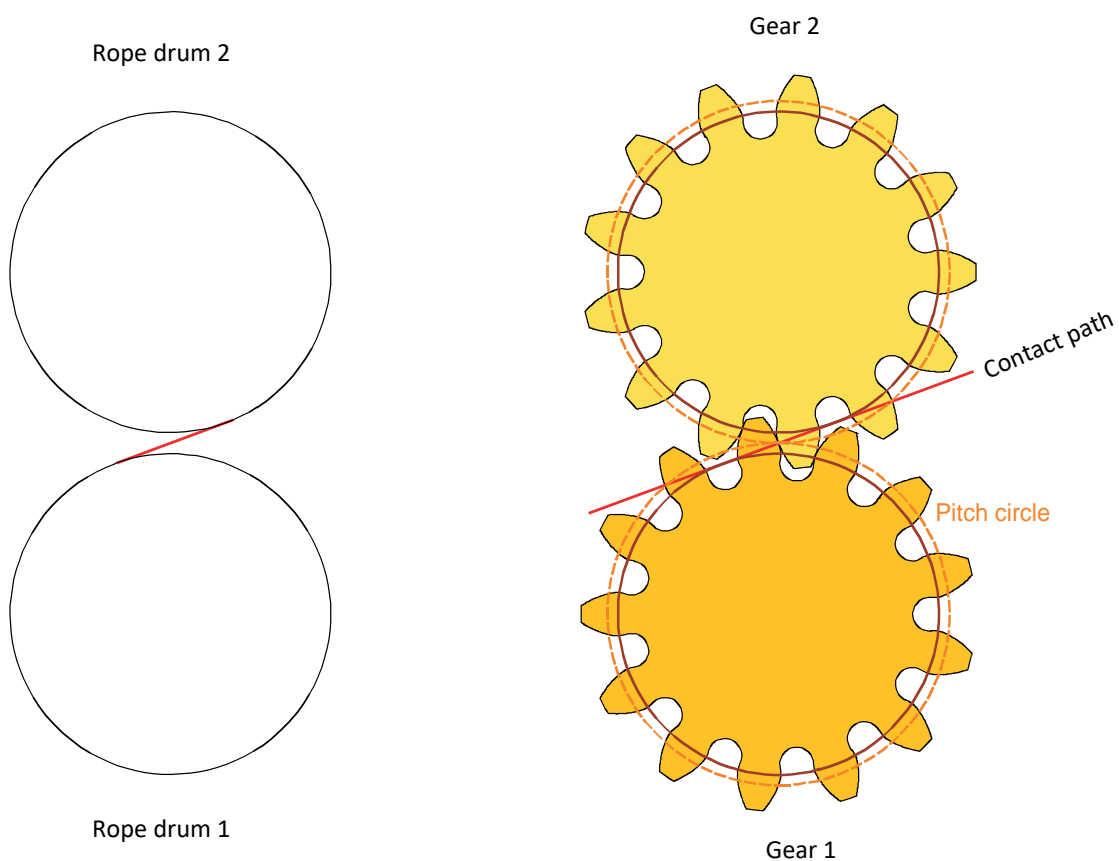


Fig. 1: Rope drive

On gears it is the involutes of the tooth flanks that are rolling off each other instead of the rope. The tangential connection of the two base circles form the contact path, along which the two involutes roll off each other. If the tooth thickness of the two gears at the pitch circle is half a pitch each, then the centre distance is the sum of the two pitch circle radii. Two gears working together must have the same pitch at the pitch circle. They must therefore also have the same module and pressure angle.

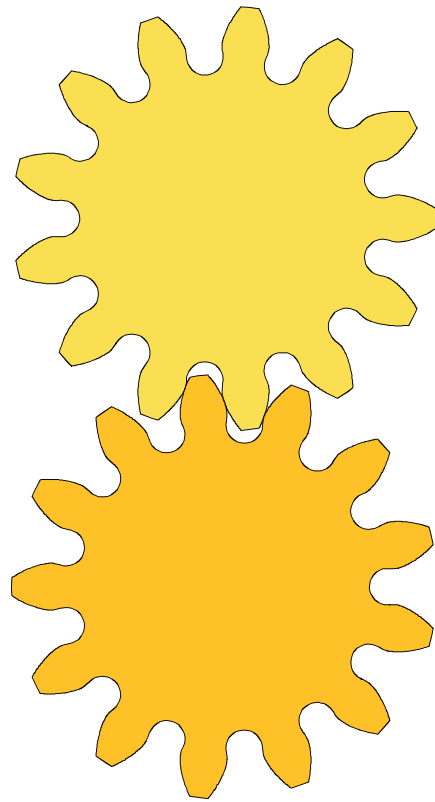


Fig. 2: Gear pair

The tooth flanks of a gear pair roll off each other along their entire tooth height. Before they separate at the end of the contact path, the subsequent tooth pair must be in mesh. If this happens at the exact same time, the contact ratio is 1. If the subsequent tooth pair comes into mesh before the previous pair has moved off the contact path, the contact ratio is greater than 1. To guarantee an even power transmission, the contact ratio must always be greater than 1. A contact ratio of 1.3 means that during 30% of the rolling time, 2 tooth pairs are in mesh. The contact ratio should ideally be between 1.2 and 1.5.

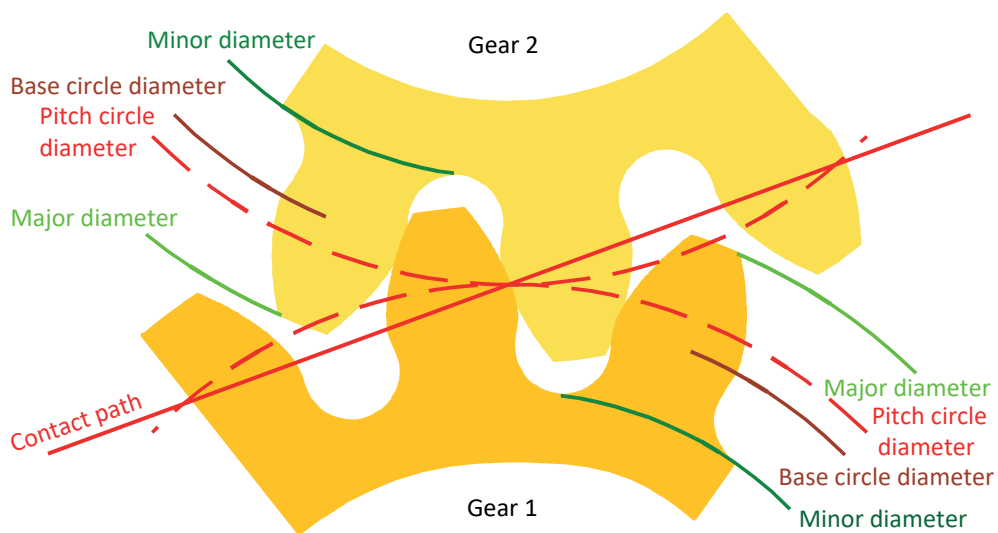


Fig. 3: Wälzvorgang zweier Stirnräder

## 1.2 Speed, centre distance, addendum modification

The transmission of torque via gears from the driving to the driven axis is usually carried out to reduce or increase the gear ratio to achieve a faster or slower speed. Both axes have bearings and a fixed centre distance. Apart from the mechanical strength, the number of teeth of both gears and the centre distance are the most important criteria. The transmission is achieved via a specific gear tooth ratio between gear 1 and gear 2. The transmission ratio is designated with  $i$ .

Number of teeth on the driving gear	$z_1$
Number of teeth on the driven gear	$z_2$
Transmission ratio	$i$

$$i = \frac{z_1}{z_2}$$

If  $i$  is greater than 1, a speed reduction will take place, if  $i$  is smaller than 1 the speed will be increased. The speed will remain the same if  $i=1$ .

The transmission ratio is the decisive factor when designing a gear pair. The smaller gear usually specifies the initial geometry because of mechanical strength requirements. The torque must be transferable from the core of the root diameter and through the tooth thickness of the tooth. Information regarding the strength calculation can be found in DIN 3950. The possible number of teeth is limited downward. If the number of teeth is too small, the contact ratio falls below 1, which means the teeth are not always in contact during the rolling process. The minimum number of teeth is usually around 12.

**The initial values are specified for gear 1 in the following example:**

The abbreviations do not correspond exactly with the DIN. Since various standards use different designations, mainly capital letters are used in this book.

Gear ratio:  $i=1.25$   
Number of teeth: 16 from strength calculations  
Module: 2.0 from strength calculations  
Pressure angle:  $20^\circ$  according to DIN 867 basic tooth profile

The tooth thickness results from the even distribution of tooth thickness and tooth space along the pitch circle.

Number of teeth	$z$
Module	$m$
Pressure angle	$\alpha$
Pitch circle diameter	$d$
Base circle diameter	$d_b$
Tooth thickness	$s$

$$d = m \times z$$

Circumference of the pitch circle diameter =  $d \times \pi$

$$\text{Arc of a pitch at the pitch circle diameter} = \frac{d \times \pi}{z}$$

$$\text{Since } D = m \times z, \text{ the arc of a pitch is } = \frac{m \times z \times \pi}{z} \text{ or } m \times \pi$$

The pitch is evenly distributed between tooth and tooth space.

$$\text{The tooth thickness } s \text{ is therefore: } s = \frac{m \times \pi}{2}$$

$$d = m \times z = 2.0 \times 16 = 32.000$$

$$s = \frac{m \times \pi}{2} = \frac{2 \times \pi}{2} = 3.1416$$

The base circle diameter is calculated from the ratio between the pitch circle diameter and the pressure angle  $\alpha$  at the pitch circle:

$$d_b = d \times \cos \alpha$$